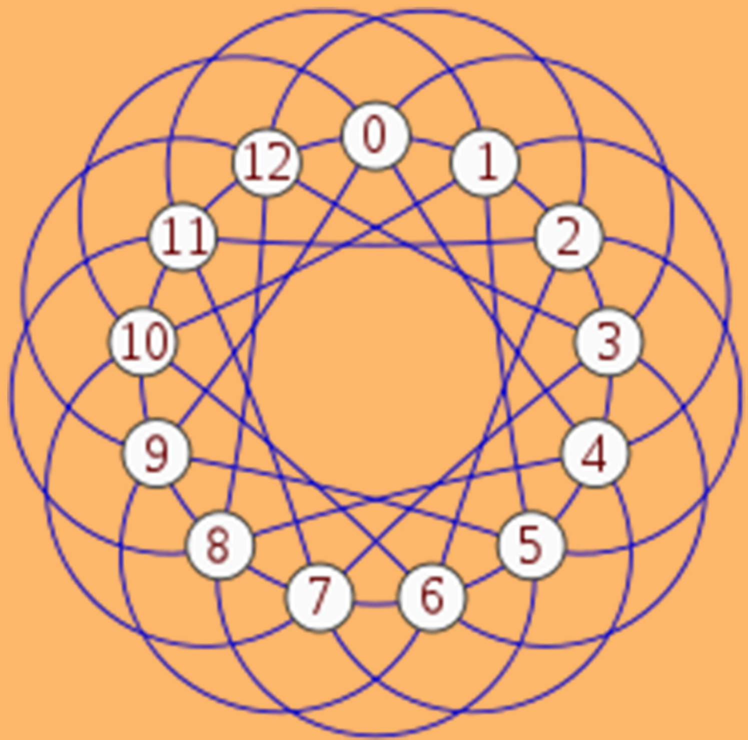
Colloquium on Combinatorial Designs

2020.12.27 8:30-12:00

https://meeting.tencent.com/s/18Qn9b0gAJiR

ID: 100 963 177



Invited Speakers

Jie Han	**Finding perfect matchings in dense hypergraphs**
Shohei Satake	**On the restricted isometry property of the Paley matrix and related results**
Shuang-qing Liu	**Constructions for constant dimension codes and related codes**

Organisers: Tao Feng, Xiande Zhang, Yue Zhou

Colloquium on Combinatorial Designs

Organized by Tao Feng, Xiande Zhang and Yue Zhou

December 27, 2020

Information

Our 2nd colloquium will be held via Tencent Voov meeting on 27th Deceomber from 8:30 to 12:00. It consists of three invited talks, each of which will take around 1 hour. There will be a 5-minutes break between every two talks.

ID: 100 963 177.

Link: https://meeting.tencent.com/s/18Qn9b0gAJi

Contents

Information **iii**

Abstracts **1**

Finding perfect matchings in dense hypergraphs (*Jie Han*) 1

On the restricted isometry property of the Paley matrix and related results (*Shohei Satake*) 1

Constructions for constant dimension codes and related codes (*Shuang-qing Liu*) 2

Abstracts

Finding perfect matchings in dense hypergraphs

27 Dec
8:30am

Jie Han

Matchings are fundamental objects in graph theory and have many applications and connections to other branches of mathematics and science. Hypergraph matchings emerge surprisingly as natural tools for many seemingly irrelevant subjects and play important roles in the recent exciting developments. In this talk we will focus on finding perfect matchings in dense hypergraphs, which has been a central topic in hypergraph matching theory for the last two decades.

On the restricted isometry property of the Paley matrix and related results

27 Dec
9:40am

Shohei Satake
Kumamoto University, Japan

Matrices with *restricted isometry property* (*RIP*) defined below have important applications to signal processing since, by adopting them, it is possible to measure and recover sparse signals using significantly fewer measurements than the dimension of the signals.

Definition 1 (Restricted isometry property, RIP) *Let Φ be a complex $M \times N$ matrix. Suppose that $K \leq M \leq N$ and $0 \leq \delta < 1$. Then Φ is said to have the (K, δ)-restricted isometry property (RIP) if*

$$(1-\delta)||\mathbf{x}||^2 \leq ||\Phi\mathbf{x}||^2 \leq (1+\delta)||\mathbf{x}||^2 \tag{1}$$

for every N-dimensional complex vector $\mathbf{x}$ with at most K non-zero entries. Here $||\cdot||$ denotes the ℓ_2 norm.

According to Candès (2008), for applications to signal processing, it suffices to investigate the (K, δ)-RIP matrix for some $\delta < \sqrt{2} - 1$. In addition, the *sparsity* K is expected to be as large as possible.

It is known that the problem checking whether a given matrix has RIP is NP-hard. Thus many publications have attempted to give deterministic constructions of matrices having RIP. Most of known constructions of $M \times N$ matrices use the coherence to estimate RIP, however, it can certify the (K, δ)-RIP with only $K = O(\sqrt{M})$, which follows from the Welch bound. This barrier for the magnitude of the order of K is called the *square-root bottleneck* or *quadratic bottleneck*. From this situation, the following problem arises.

Problem 2 *Construct an* $M \times N$ *matrix* Φ *having the* (K,δ)*-RIP with* $K = \Omega(M^\gamma)$ *for some* $\gamma > 1/2$ *and* $\delta < \sqrt{2} - 1$.

To our best knowledge, the first (unconditional) solution to this problem was given by Bourgain, Dilworth, Ford, Konyagin and Kutzarova (2011), and later was generalized by Mixon (2015). It has been conjectured that the *Paley matrix*, a $(p+1)/2 \times (p+1)$ matrix defined by quadratic residues modulo an odd prime p, satisfies the (K,δ)-RIP with $K \geq C_\delta \cdot p/\text{polylog}\, p$ for some $C_\delta > 0$ depending only on δ. Under a number-theoretic conjecture, Bandeira, Mixon and Moreira (2017) proved that when $p \equiv 1 \pmod 4$, the Paley matrix has the $(\Omega(p^\gamma), o(1))$-RIP for some $\gamma > 1/2$, which provides a conditional solution to Problem 2.

In this talk, assuming that the widely-believed Paley graph conjecture, we first show that the Paley matrix is a solution to Problem 2 for any sufficiently large prime p, including primes $p \equiv 3 \pmod 4$. Second, corresponding to a result by Bandeira, Mixon and Moreira (2017) estimating the clique number of the Paley graph, we prove that the RIP of the Paley matrix implies a new upper bound on the size of transitive subtournaments (i.e. ones with no directed cycles) in the *Paley tournament*. The bound here is significantly better than the existing bounds by Tabib (1986), Momihara and Suda (2017) for this tournament.

This talk is based on arXiv:2011.02907.

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27 Dec
10:50am

Constructions for constant dimension codes and related codes

Shuang-qing Liu

Suzhou University of Science and Technology, China

Constant dimension codes, as special subspace codes, have received a lot of attention due to its application in random network coding. The talk will summarize some important constructions for constant dimension codes: lifted MRD code, multilevel construction, parallel construction, parallel multilevel construction, and linkage construction. The talk will also introduce some relevant codes such as Ferrers diagram rank-metric codes and rank-metric codes with given ranks.
